Effort Complementarity and Sharing Rules in Group Contests

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Abstract

In this paper, we consider a prize-sharing rule design problem in a group contest with effort complementarities within groups by employing a CES effort aggregator function. We derive the conditions for a monopolization rule that dominates an egalitarian rule if the objective of the rule design is to maximize the group's winning probability. We find conditions under which the monopolization rule maximizes the group's winning probability, while the egalitarian rule is strictly preferred by all members of the group. Without effort complementarity, there cannot be such a conflict of interest.

Keywords: group contest, complementarity in efforts, free riding, prize-sharing rule

JEL Classification Numbers: C72, D23, D74

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is identical to that where a group's winning probability goes down as the population of the standpoint of the group's winning probability. This happens when (i) there exists moderate effort complementarity and (ii) the marginal effort cost is moderately concave. This result cannot be obtained without effort complementarity.

The remainder of the paper is organized as follows. Section 2 presents our model. In Section 3, we show that the results in Esteban and Ray (2001) and Nitzan and Ueda (2014) still extend in the presence of effort complementarity. Section 4 shows our main result that a group leader and his/her group members may have a conflict of interest with effort complementarity. Section 5 explains how our results for Nash equilibrium in the intragroup game can be extended to an equilibrium analysis in a group contest game, and proves the existence and uniqueness of the equilibrium. Section 6 concludes by discussing the importance of effort complementarity and commenting on a model in Epstein and Mealem (2009), the details of which are given in Appendix B. All proofs are collected in Appendix A.

2 The Model

We consider a contest in which $m \geq 2$ groups compete for a prize, focusing on a representative group $i = 1, 2, \ldots, m$. The population of group i is denoted by $n_i \geq 1$. Group members choose their effort levels e_{ij}, $j = 1, 2, \ldots, n_i$, which contribute to their group's winning probability, simultaneously and non-cooperatively. Group members' efforts are aggregated by the CES function of $X_i = (\sum_{j=1}^{n_i} e_{ij}^r)^{\frac{1}{r}}$, where $r \in (0, 1]$ indicates the degree of the effort complementarity.[5] This CES aggregator function becomes a linear function (perfect substitutes—no effort is also zero).

The winning probability of group i is described as a contest success function[6] $P_i = \frac{X_i}{X_i + X_{-i}}$, where $X_{-i} = \sum_{k \neq i} X_k$ is the other groups' aggregated effort levels. The prize comprises divisible private goods that are shared among members of the winning group, and the value of the prize is normalized to 1. We denote the share of member j in group i by $a_{ij} \in [0; 1]$ and group i's (prize) sharing rule by $a_i = (a_{i1}; \dots ; a_{in_i})$ with $\sum_{j=1}^{n_i} a_{ij} = 1$. The group leader cannot observe each member's effort or an aggregated group effort.[7] We assume that group i's prize-sharing rule is chosen by the group leader before each member decides his/her effort level. The effort cost function is common to all members with a constant elasticity $\alpha \geq 1$; i.e., member j's effort cost in group i is described by $\frac{1}{\alpha} e_{ij}^{\alpha}$. We assume that at least either $r \leq 1$ or $\alpha \geq 1$ is a strict inequality. The expected payoff for member j in group i is $U_{ij} = P_i a_{ij} - \frac{1}{\alpha} e_{ij}^{\alpha}$. We assume that all of the above is common knowledge among all players.

Each member in a group decides his/her effort level to maximize his/her expected payoff. We assume that group i members regard the other groups' aggregate effort X_{-i} as given and consider a Nash equilibrium of group i's effort contribution game as their best response to the other groups' aggregate effort X_{-i}. Then, the first-order condition of any member j in group i is

$$\frac{\partial U_{ij}}{\partial e_{ij}} = \frac{\left(\sum_{j=1}^{n_i} e_{ij}^r\right)^{\frac{1}{r}-1} e_{ij}^{r-1} X_{-i}}{\left(\left(\sum_{j=1}^{n_i} e_{ij}^r\right)^{\frac{1}{r}} + X_{-i}\right)^2} a_{ij} - e_{ij}^{\alpha - 1} = 0.$$

This can be rewritten as

$$P_i(1 - P_i)\frac{e_{ij}^r}{X_i^r} a_{ij} - e_{ij}^{\alpha} = 0. \qquad (1)$$

With these first-order conditions (1), we can investigate how the sharing rule a_i affects the members' equilibrium effort levels $(e_{i1}; \dots; e_{in_i})$ in an effort contribution game in group i.

[6] We employ the Tullock-form contest success function (Tullock 1980).

[7] Nitzan and Ueda (2011) assume that individual effort levels are observable by the group leader and analyze

From (1), we have $e^{r}_{ij} = \frac{P_i(1 \quad P_i)}{X^{r}_{i}}^{\frac{r}{\quad r}} a_{ij}^{\frac{r}{\quad r}}$. Summing up all e^{r}_{ij} in group i, we have

$$X^{r}_{i} =$$

$a_{il} = 0$ for any other member $l \neq j$). The next proposition shows that these two rules maximize the winning probability of group i, depending on r and .

Proposition 1. *When* $2r <$ *, the egalitarian rule maximizes the winning probability of group* i*. When* $2r >$ *, the monopolization rule maximizes the winning probability of group* i*. When* $2r =$ *, the winning probability of group* i *is the same under any sharing rules.*

We can interpret this result in the context of R&D competition. Some R&D projects benefit from coordinated efforts (strong effort complementarity: small r), while others do not (weak effort complementarity: large r). Proposition 1 says that the group leader should choose the egalitarian rule for projects with strong effort complementarity ($2r <$), since treating everybody equally enhances aggregate effort the most. In contrast, the group leader should use the monopolization rule by selecting a single member for projects with weak complementarity ($2r >$), since it eliminates all free-riding incentives and maximizes an incentive for effort by letting the selected member monopolize the prize. If $2r =$, then A_i is the same under any sharing rules $(a_{ij})_{j=1}^{n_i}$. Nitzan and Ueda (2014) report the above result without effort complementarity ($r = 1$; Proposition 4 in Nitzan and Ueda 2014).

The probabilities under the egalitarian rule and the monopolization rule are denoted by P_{iE} and P_{iM}, respectively. Under the egalitarian rule in group i, every member's effort is the same, which is denoted by e_i in a Nash equilibrium in group i. Then $X_i = (\sum_{j=1}^{n_i} e_{ij}^r)^{\frac{1}{r}} = (n_i e_i^r)^{\frac{1}{r}} = n_i^{\frac{1}{r}} e_i$. Thus, (1) becomes

$$e_i = P_{iE}(1 \quad P_{iE})\frac{1}{n_i^2}. \tag{3}$$

Since (2) implies $e_i = n_i^{\frac{2}{\beta}} P_{iE}^{\frac{1}{\beta}}(1 \quad P_{iE})^{\frac{1}{\beta}}$, when we substitute this into the definition of P_i, we

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are able to solve P_{iE} as a function of parameters, in particular, n_i and r implicitly:

$$P_{iE}(n_i;r) = \frac{n_i^{\frac{1}{r}} e_i}{n_i^{\frac{1}{r}} e_i + \ X \quad _i} = \frac{n_i^{\frac{\beta \quad 2r}{r\beta}} P_{iE}(n_i;r)^{\frac{1}{\beta}} (1 \quad P_{iE}(n_i;r))^{\frac{1}{\beta}}}{n^{\beta \quad 2r}}$$

group i, despite the fact that the monopolization rule achieves a higher winning probability than the egalitarian rule. In contrast, if $1 - \frac{n_i}{n_i+1} > P_{iM}$ *is not satisfied, then the single member who monopolizes the prize prefers the monopolization rule to the egalitarian rule.*

This Pareto dominance of the egalitarian rule is due to the complementarity among group members' efforts. Without the complementarity, this Pareto dominance disappears. In fact, at $r = 1$, the Pareto dominance does not hold.

5 Equilibrium in Group Contest

We can apply our analysis to show that our group contest model has an equilibrium. We will consider a two-stage game as follows. Stage 1: Each group leader who maximizes the winning probability of his/her group decides its sharing rule simultaneously, and Stage 2: members of all groups simultaneously choose their effort levels. In this paper, we assume that each groups' sharing rules are observable and employ subgame perfect equilibrium as our solution concept.[12] We can allow for asymmetric groups—different groups can have different ${}_i$, r_i, and n_i. The key is to show that a Nash equilibrium exists and is unique in Stage 2. We show that each group's best response to the aggregation of the other groups' effort levels X_{-i} is at a Nash equilibrium. The effort contribution game of any group i in Stage 2 is described as a function ${}_i(X_{-i}; a_i; {}_{-i}; r_i) \to X_i$. Using the share-function approach (Esteban and Ray 2001, Ueda 2002, and Cornes and Hartley 2005),[13] we can guarantee the existence and uniqueness of the Nash equilibrium by each ${}_i$'s continuity and strict monotonicity in X_{-i}.

[12] Readers may think that it is unrealistic to assume that groups can observe other groups' sharing rules. Nitzan and Ueda (2011) assume that sharing rules are the private information of each group and use perfect Bayesian equilibrium with the same beliefs for other groups' sharing rules at every information set. Since the model does not involve a real asymmetric information problem, their perfect Bayesian equilibrium coincides with our subgame perfect equilibrium under complete information.

[13] We thank Kaoru Ueda for suggesting that we use the share function approach.

This implies that each group leader's objective is to maximize his/her A_i in Stage 1, which is the same result as Lemma 1.

Lemma 5. *In Stage 1 of the group contest game, the equilibrium winning probability of group i is increasing in A_i. That is, group i's winning probability is maximized by a sharing rule $(a_{ij})_{j=1}^{n_i}$ that maximizes A_i.*

This lemma leads us to a counterpart of Proposition 1|that is, the results of Proposition 1 are valid in the two-stage group contest game.

Proposition 5. *In Stage 1 of the group contest game, each group i's leader chooses its sharing rule to maximize the winning probability P_i as follows: (i) use the egalitarian rule if $2r_i < \ _i$, (ii) use the monopolization rule* [14] *if $2r_i > \ _i$, and (iii) use any sharing rule if $2r_i = \ _i$.*

A corollary of this proposition is that there is an essentially unique subgame perfect equilibrium in our group contest game, since each group leader's strategy is solely dependent on $2r_i \lesseqgtr \ _i$.

Corollary 1. *For all $(r_i; \ _i)_{i=1}^{m}$, groups' equilibrium winning probabilities $(P_i)_{i=1}^{m}$ are uniquely determined.*

The results of Proposition 5 depend only on the exogenous variables of r_i and $\ _i$. Thus, Proposition 5 and Corollary 1 indicate that Proposition 3 is also valid at the subgame perfect equilibrium in the two-stage group contest game. That is, for some group i, when $2r_i > \ _i$ and $1 + \frac{1}{n_i} > \ _i$ under the asymmetric parameters, if $1 \quad \frac{n_i}{n_i+1} \ _i > P_{iM}$ holds, then there is a con ict of interest between the group leader and his/her group members at the subgame perfect equilibrium. If $1 \quad \frac{n_i}{n_i+1} \ _i > P_{iM}$ is violated, then the monopolizing member has an incentive to work with the group leader, since it is in their common interests to choose the

[14] When the group leader chooses the monopolization rule at Stage 1, e ort complementarity is irrelevant on the equilibrium path. E ort complementarity is in e ect only o the equilibrium path.

monopolization rule and exclude the rest of the group. In addition, if the parameters are symmetric, the condition is simply described as the relation among the number of groups, the group population, and the elasticity of the marginal effort cost. In this case, since e_{iM} becomes $\frac{1}{m}$, the condition is $1 - \frac{n_i}{n_i+1} > \frac{1}{m}$.

6 Concluding remarks

We conclude our paper by commenting on Epstein and Mealem (2009). They use a generalized Tullock contest by introducing power $r \in [0, 1]$: i.e., $X_i = \sum_{j=1}^{n_i} e_{ij}^r$. This form may look similar to our CES form, $X_i = \left(\sum_{j=1}^{n_i} e_{ij}^r\right)^{\frac{1}{r}}$, and readers may wonder if our Proposition 3 may hold in their case. It turns out that their generalized Tullock contest cannot generate conflicts of interest between the group leader and his/her group members—we can confirm that with their form, the egalitarian rule's Pareto dominance in Proposition 3 cannot occur. Thus, effort complementarity is essential in getting our conflict-of-interest result. In contrast, with their contest success function, our Propositions 1 and 2 hold. We detail the analysis in Appendix B.

Appendix A

Here, we collect all proofs.

Proof of Lemma 1. Recalling $X_i = (\sum_{j=1}^{n_i} e_{ij}^r)^{\frac{1}{r}}$ and given $X \quad {}_i$, maximizing the winning probability of group i means that X_i becomes as large as possible at Nash equilibrium in group i. If X_i is a strictly increasing function of A_i, we can maximize X_i by maximizing A_i subject to $\sum_{j=1}^{n_i} a_{ij} = 1$.

From (2), let $\phi(X_i; A_i) = X_i \quad P_i(1 \quad {}_i$

d

for $> r$ by using $A_i = \frac{X_i^{\beta}}{P_i(1\ \ P_i)}$ from (2). Thus, X_i is a strictly increasing function in A_i for $> r$. ■

Proof of Proposition 1. From Lemma 1, it is enough to maximize A_i. It is also enough to maximize the contents in parentheses in A_i because $\frac{\ \ r}{r} > 0$. Note that $A_i^{\frac{r}{\beta\ \ r}} = \sum_{j=1}^{n_i} a_{ij}^{\frac{r}{\beta\ \ r}}$ is an additively separable function. Since $r > 0$, our maximization problem boils down to

$$\max \sum_{j=1}^{n_i} a_{ij}^{\frac{r}{\beta\ \ r}} \text{ subject to (i) } \sum_{j=1}^{n_i} a_{ij} = 1 \text{ and (ii) } a_{ij} \quad 0 \text{ for all } j = 1;\ldots;n_i.$$

Thus, it is easy to see that $\frac{\ \ r}{r} \lesseqgtr 1$ dictates the optimal sharing rule. We obtain three cases:

Case 1: If $2r <$, A_i is maximized when $a_1 = a_2 = \cdots = a_{n_i} = 1/n_i$.

Case 2: If $2r =$, A_i is constant for any sharing rule.

Case 3: If $2r >$, A_i is maximized when $a_{ij} = 1$ for a single j, and $a_{i\ell} = 0$ for all other ℓ.

■

Proof of Lemma 2. Let $n_i^{\frac{\beta\ \ 2r}{r\beta}}$ in relation to n_i in (4). Rewriting (4), we have

$$(1\quad P_{iE})(P_{iE}(1\quad P_{iE}))^{\frac{1}{\beta}} = P_{iE} X\ \ _i\ : \tag{10}$$

By totally differentiating the above, we obtain

$$(P_{iE}(1\quad P_{iE}))^{\frac{1}{\beta}}\quad X\ \ _i + \frac{1}{\ }(1\quad P_{iE})(P_{iE}(1\quad P_{iE}))^{\frac{1}{\beta}\ \ 1}(1\quad 2P_{iE})\ dP_{iE} = P_{iE} X\ \ _i d\ :$$

After solving (10) for $X\ \ _i$

and

$$\frac{d}{dr} = \frac{1}{r^2}(\log n_i) \quad > 0;$$

respectively. We obtain the results using the chain rul■

Proof of Lemma 3. Recall $U_{iE}(n_i;r) = \frac{P_{iE}(n_i;r)}{n_i}\Big(1 \quad \frac{1}{}(1 \quad P_{iE}(n_i;r))\ \frac{1}{n_i} \qquad \bar{U}(n_i;P_{iE}(n_i;r))$.

This implies

$$\frac{@\bar{U}}{@P_{iE}} = \frac{1}{n_i} \quad \frac{1}{n_i^2}(1 \quad 2P_{iE}) = \frac{1}{n_i^2}(n_i \quad + 2P_{iE} \quad 1): \qquad (11)$$

Thus, by totally di erentiating $\bar{U}(n_i;P_{iE}(n_i;1))$ with respect to n_i using (6), we obtain

$$\frac{dU_{iE}(n_i;1)}{dn_i}$$

$$= \frac{@\bar{U}}{@n_i} + \frac{@\bar{U}}{@P_{iE}}\frac{dP_{iE}}{dn_i}$$

$$= \frac{P_{iE}}{n_i^2} \quad 1 + \frac{2}{n_i}(1 \quad P_{iE}) \quad + \frac{(2 \qquad)P_{iE}(1 \quad P_{iE})}{n_i(1 \quad 2P_{iE} \qquad)} \quad \frac{1}{n_i^2}(n_i \quad + 2P_{iE} \quad 1)$$

$$= \frac{P_{iE}}{n_i^3} \quad n_i + \frac{2}{}(1 \quad P_{iE}) + (1 \quad P_{iE})\frac{2}{1 \quad 2P_{iE}} \quad n_i + \frac{1}{}(2P_{iE} \quad 1)$$

$$= \frac{P_{iE}}{n_i^3 \ (1 \quad 2P_{iE} \qquad)}$$

$$[\ n_i \ (1 \quad 2P_{iE} \qquad) + 2(1 \quad P_{iE})(1 \quad 2P_{iE} \qquad) + (1 \quad P_{iE})(2 \qquad)(\ n_i \quad (1 \quad 2P_{iE}))]:$$

Since $1 \quad 2P_{iE} \qquad < 0$, we can focus on the sign of the contents of the brackets:

$$[\ \ ] = \ n_i \ (1 \quad 2P_{iE} \qquad) + \ n_i \ (2 \qquad 2P_{iE} + P_{iE} \ ) \ (1$$

$$+2(1 \quad 2P_{iE} \qquad)(1 \quad P_{iE}) \quad (1 \quad P_{iE})(2 \qquad)(1 \quad 2P_{iE})$$

$$= \quad [($$

Consider the case of $r = \frac{}{2}$. Since $P_{iE}(n_i; \frac{}{2}) = P_{iE}(1; \frac{}{2}) = P_{iM}$ by Proposition 1, we have

$$U_{iE}(n_i; \tfrac{}{2}) = \frac{P_{iE}(n_i; \frac{}{2})}{n_i}\left(1 - \frac{1}{\ }(1 - P_{iE}(n_i; \tfrac{}{2}))\frac{1}{n_i}\right)$$
$$= \frac{P_{iM}}{n_i}\left(1 - \frac{1}{\ }(1 - P_{iM})\frac{1}{n_i}\right):$$

By subtracting $U_{iM} = P_{iM}\left(1 - \frac{1}{\ }(1 - P_{iM})\right)$ from $U_{iE}(n_i; \frac{}{2})$, we obtain

$$U_{iE}(n_i; \tfrac{}{2}) - U_{iM} = P_{iM}\left(-1 + \frac{1}{\ }(1 - P_{iM}) + \frac{1}{n_i}\left(1 - \frac{1}{\ }(1 - P_{iM})\frac{1}{n_i}\right)\right)$$
$$= P_{iM}\left(-\left(1 - \frac{1}{n_i}\right) - \frac{1}{\ }\left(1 - \frac{1}{n_i^2}\right) - \frac{1}{\ }P_{iM}\left(1 - \frac{1}{n_i^2}\right)\right)$$
$$= P_{iM}\left(1 - \frac{1}{n_i^2}\right)\left(-\frac{n_i}{n_i+1} + \frac{1}{\ } - \frac{1}{\ }P_{iM}\right):$$

Then, the condition of $U_{iE}(n_i; \frac{}{2}) > U_{iM}$ is

$$1 - \frac{n_i}{n_i+1} > P_{iM}: \qquad (12)$$

That is, if (12) is satisfied, $U_{iE}(n_i; \frac{}{2}) > U_{iM}$ holds, while $U_{iE}(n_i; 1) < U_{iM}$. Since $\frac{dP_{iE}}{dr} < 0$ holds by (5) in Lemma 2 and from (11), $\frac{@U}{@P_{iE}} = \frac{1}{n_i^2}(n_i - + 2P_{iE} - 1) > 0$, we have $\frac{dU_{iE}(P_{iE}(r))}{dr} = \frac{@U_{iE}}{@P_{iE}}\frac{dP_{iE}}{dr} < 0$, which is U_{iE} monotonically decreasing in r. Considering the above facts and given that U_{iE} is continuous in r, there is a unique $\hat{r}$ 2 $(\frac{}{2}; 1)$, such that $U_{iE}(n_i; r) < U_{iM}$ holds for all r 2 $(\hat{r}; 1]$ and $U_{iE}(n_i; r) > U_{iM}$ holds for all r 2 $[\frac{}{2}; \hat{r})$. ■

Proof of Lemma 5. First, focus on the $P_i(X; A_i)$ function. Starting from the original A_i and equilibrium X , A_i is increased by $A_i > 0$. Since $\frac{@P_i}{@A_i} > 0$ for all X from (9), the P_i function shifts up vertically. Let $\bar{X}$ be such that $P_i(X ; A_i) = P_i(\bar{X}; A_i + A_i)$ (see Figure 1). Since $\frac{@P_i}{@X} < 0$ from (8), $\bar{X} > X$ holds, and for any X 2 $(X ; \bar{X})$, we have $P_i(X; A_i + A_i) > P_i(X ; A_i)$. Recall that the equilibrium X is described by the aggregate share function

$$f(X ; A) = \sum_{i^0 6= i} P_{i^0}(X ; A_{i^0}) + P_i(X ; A_i) = 1:$$

Let A_{i} be a vector that removes A_i from A. By increasing A_i by A_i, the equilibrium aggregate effort X satisfies

$$f(X ; A_i + A_i; A_{i}) = \sum_{i^0 6= i} P_{i^0}(X ; A_{i^0}) + P_i(X ; A_i + A_i) = 1:$$

Since $\frac{\partial P_{i'}}{\partial X} < 0$ for all $i' = 1, \ldots, m$, we have $\bar{X} > X^{*}$ and

$$f(\bar{X}; A_i + \Delta A_i; A_{-i}) = \sum_{i' \neq i} P_{i'}(\bar{X}; A_{i'}) + P_i(\bar{X}; A_i + \Delta A_i)$$
$$= \sum_{i' \neq i} P_{i'}(\bar{X}; A_{i'}) + P_i(X^{*}; A_i) < 1.$$

By the intermediate value theorem, $X^{**} \in (X^{*}, \bar{X})$ holds. We conclude $P_i(X^{**}; A_i + \Delta A_i) > P_i(X^{*}; A_i)$.

This implies that as A_i increases, $P_i(X^{*}; A_i)$ increases. That is, maximizing A_i achieves the maximum winning probability for group i. ■

Appendix B

Here, we repeat our analysis by using the Epstein and Mealem's generalized Tullock contest, and show that Lemma 1 and Proposition 1 hold. We confirm this first. The expected payoff of member j in group i is $U_{ij} = \frac{\sum_{j=1}^{n_i} e_{ij}^r}{\sum_{j=1}^{n_i} e_{ij}^r + X_{-i}} a_{ij} - \frac{1}{\beta} e_{ij}^{\beta}$. The first order condition is

$$\frac{\partial U_{ij}}{\partial e_{ij}} = \frac{r e_{ij}^{r-1} X_{-i}}{\left(\sum_{j=1}^{n_i} e_{ij}^r + X_{-i}\right)^2} a_{ij} - e_{ij}^{\beta - 1} = 0.$$

This can be rewritten as

$$P_i(1 - P_i) \frac{r e_{ij}^r}{\sum_{j=1}^{n_i} e_{ij}^r} a_{ij} - e_{ij}^{\beta} = 0. \tag{13}$$

We process a procedure similar to the one at the end of Section 2 and get $e_{ij}^r = \left(\frac{rP_i(1-P_i)}{X_i}\right)^{\frac{r}{\beta - r}} a_{ij}^{\frac{r}{\beta - r}}$ from (13). By summing up each e_{ij}^r, we have $\sum_{j=1}^{n_i} e_{ij}^r = X_i = \left(\frac{rP_i(1-P_i)}{X_i}\right)^{\frac{r}{\beta - r}} \hat{A}_i$ where $\hat{A}_i = \sum_{j=1}^{n_i} a_{ij}^{\frac{r}{\beta - r}}$. Let $\phi(X_i; \hat{A}_i) = X_i - \left(\frac{rP_i(1-P_i)}{X_i}\right)^{\frac{r}{\beta - r}} \hat{A}_i = 0$. By differentiating ϕ with respect to $\hat{A}_i$ and noting that P_i is a function of X_i, we have

$$\frac{dX_i}{d\hat{A}_i} = -\frac{\phi_{A_i}}{\phi_{X_i}} = \frac{(rP_i(1-P_i)/X_i)^{\frac{r}{\beta - r}}}{(\beta - r + 2rP_i)/(\beta - r)} > 0$$

for $\beta > r$ by using $\hat{A}_i = X_i \left(\frac{rP_i(1-P_i)}{X_i}\right)^{-\frac{r}{\beta - r}}$. Therefore, since Lemma 1 holds, Proposition 1 also holds in this case. Proposition 2 holds as well. However, Proposition 3 does not hold.

We check this second. Under the egalitarian rule, since $X_i = \sum_{j=1}^{n_i} e_{ij}^r = n_i e_i^r$, we have $e_i = n_i^{\frac{2}{}} r^{\frac{1}{}} P_{iE}^{\frac{1}{}} (1 \quad P_{iE})^{\frac{1}{}}$ from (13). Using this, we have

$$P_{iE} = \frac{n_i e_i^r}{n_i e_i^r + X_{\ i}} = \frac{n_i^{\frac{2r}{}} r^{\frac{r}{}} P_{iE}^{\frac{r}{}} (1 \quad P_{iE})^{\frac{r}{}}}{n_i^{\frac{2r}{}} r^{\frac{r}{}} P_{iE}^{\frac{r}{}} (1 \quad P_{iE})^{\frac{r}{}} + X_{\ i}} \qquad (14)$$

and

$$U_{iE} = P_{iE} \frac{1}{n_i} \quad \frac{1}{} e_i = \frac{P_{iE}}{n_i} \quad 1 \quad \frac{1}{}(1 \quad P_{iE}) \frac{r}{n_i} \quad :$$

Let $\hat{} \quad n_i^{\frac{2r}{}}$ in relation to n_i in (14). We process the same procedure as in the proof of Lemma 2. Rewriting (14), we have

$$r^{\frac{r}{}} (1 \quad P_{iE}) (P_{iE} (1 \quad P_{iE}))^{\frac{r}{}} = P_{iE} X_{\ i} \hat{} : \qquad (15)$$

By totally di erentiating the above expression and conducting the same operations as the proof of Lemma 2, we obtain

$$\frac{dP_{iE}}{d\hat{}} = \frac{h}{\hat{}} \frac{P_{iE} (1 \quad P_{iE})}{1 + \frac{r}{}}$$

However, this condition contradicts the definition of the probability. Lemma 4 does not hold.

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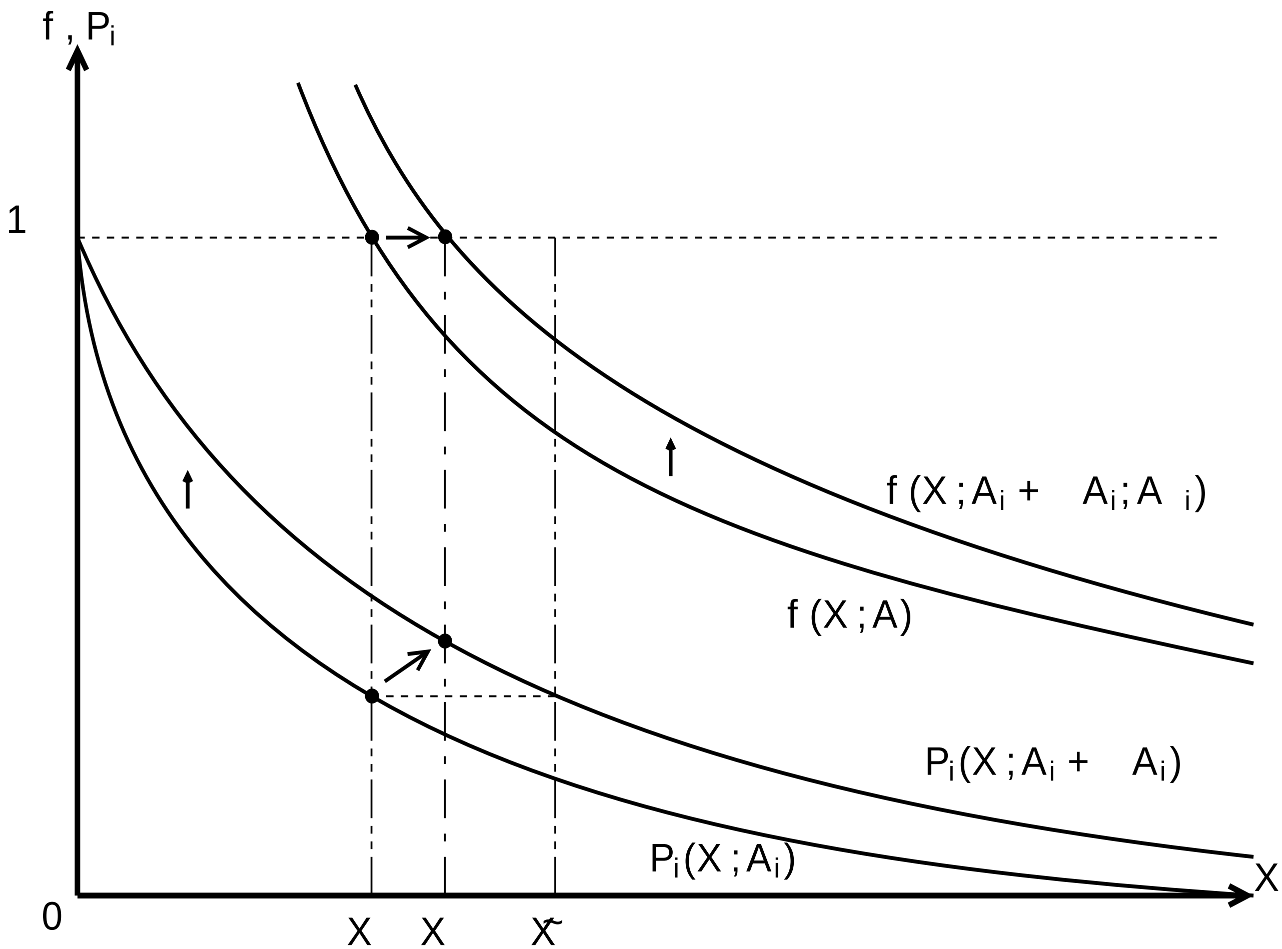

Figure 1: Share function $P_i(X;A_i)$ of group i and aggregate share function $f(X;A)$